NAG Toolbox for MATLAB

f07je

1 Purpose

f07je computes the solution to a real system of linear equations $AX = B$, where A is an n by n symmetric positive-definite tridiagonal matrix and X and B are n by r matrices, using the LDL^{T} factorization returned by f07jd.

2 Syntax

```
[b, info] = f07je(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07je should be preceded by a call to f07jd, which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. f07je then utilizes the factorization to solve the required equations. Note that the factorization may also be regarded as having the form $U^{\mathrm{T}}DU$, where U is a unit upper bidiagonal matrix.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

5 Parameters

5.1 Compulsory Input Parameters

1: **d**$(*)$ – **double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

Must contain the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

2: **e**$(*)$ – **double array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n - 1)$ subdiagonal elements of the unit lower bidiagonal matrix L. **e** can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^{\mathrm{T}}DU$ factorization of A.

3: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r matrix of right-hand sides B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **d**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **n**, 2: **nrhs_p**, 3: **d**, 4: **e**, 5: **b**, 6: **ldb**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Following the use of this function f07jg can be used to estimate the condition number of A and f07jh can be used to obtain approximate error bounds.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr.

The complex analogue of this function is f07js.

9 Example

```
d = [4;
     9;
     25;
     16;
     1];
e = [-0.5;
     -0.6666666666666666;
     0.6;
     0.5];
b = [6, 10;
     9, 4;
     2, 9;
     14, 65;
     7, 23];
[bOut, info] = f07je(d, e, b)
```

```
bOut =
    2.5000    2.0000
    2.0000   -1.0000
    1.0000   -3.0000
   -1.0000    6.0000
    3.0000   -5.0000
info =
           0
```